## Proposals for Ph.D and M.Sc thesis

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I'm currently working on several fields. My main interest is related to *optimal mass transportation* (Monge-Kantorovich problem) and its applications to kinetic theory, co-agulation fragmentation models and optics. In addition, I'm also interested in chemo-taxis models, reduced dynamics for MEMS (microelectromechanical systems) and variational methods in vision.

Below I enclose some possible subjects for research students

1. Adhesive particle dynamics: The object here is to define an appropriate weak solution to the system

$$\rho_t + (u\rho)_x = 0$$
  $(\rho u)_t + (\rho u^2)_x = \rho F(x, u, t)$ 

on the real line, which represents the dynamics of particles following the force field F and undergo sticking collisions while preserving their momentum.

In their celebrated paper, Rykov, Sinai and E. proved the existence of "sticking particle solutions" to this system subjected to zero force  $F \equiv 0$ . The extension for  $F \not\equiv 0$  is nontrivial, but a traceable one. It can be the subject to a good M.Sc thesis, or the starting point for a Ph.D thesis.

- 2. Optimal Transport in optics (with Prof. Koby Rubinstein): The problem of optimal mass transportation (Monge-Kantorovich) is the source of many theoretical investigations and applications. Recently, we found new applications of this theory in optics. In particular, the formulation and calculation of optimal illumination and applications to phase detectors. There are many open problems-including the extension of this theory to singular wavefronts (caustics), as well as the need to develop efficient algorithms.
- 3. **Reduced dynamics for MEMS**: The dynamics of MEMS is represented by PDE's and appropriate boundary conditions. The methods to handle these models is by reduction to ODE in time and obtain information from both numerical and analytical investigation of these systems. The standard, first step in this direction is to idealize the MEMS devise as rigid bodies, whose finite number of degrees of freedom are used in this reduction.

It is possible, however, to include also some degrees of freedoms due to deformations of the rigid bodies in the framework of ODE. There are some important questions still awaiting for resolution. In particular, what is the optimal choice of these deformation modes, calculations of natural frequencies of the system, behavior under time periodic electrostatic forcing (parametric resonance) and more. 4. Blow-up of some chemotaxis limits: One of the models of chemotaxis is given by a system of drift-diffusion equations on 2-dimensional domain, subjected to natural boundary and initial conditions which preserve to total cell population  $M = \int \rho(x, t) dx$ :

$$\alpha \frac{\partial \rho}{\partial t} = \nabla \cdot \left( \rho \nabla U + \sigma \nabla \rho \right) \quad ; \quad (1 - \alpha) \frac{\partial U}{\partial t} = \Delta U + \rho \; .$$

Here  $\rho$  stands for the concentration of micro-organisms (cells), U is the concentration of a chemical substance produced by these cells and  $\alpha \in (0, 1)$ .

The limit  $\alpha = 1$  was considered by many authors and have a rich history. The case  $\alpha = 0$  is much less known. The system is then reduced to the heat equation with non-local source:

$$\frac{\partial U}{\partial t} = \Delta U + M \frac{e^{U/\sigma}}{\int e^{U/\sigma}} \; .$$

One very interesting question is the existence of finite time blow-up for this equation, if M is large enough.