Let $\chi, \chi' \in \hat{N}$, $\chi, \chi' \neq 1$, $\chi_{|\Gamma\cap N} = \chi'_{|\Gamma\cap N} = 1$

If $\gamma \in \Gamma \cap P s N$ the set

$$\Xi_\gamma = \{\gamma' \in \Gamma \cap P s N : m_{\gamma'} a_{\gamma'} = m_\gamma a_\gamma\}$$

contains finitely many double cosets $\Gamma_N \gamma' \Gamma_N$

We define

$$S(\gamma) = S(\chi, \chi'; \gamma) = \sum_{\delta \in \Xi_\gamma} \chi(n_1(\delta)) \chi'(n_2(\delta))$$

a <u>generalized</u> <u>Kloosterman</u> sum. Since $S(\gamma)$ depends on $\xi_\gamma = m_\gamma a_\gamma \in MA$ we may write $S(\xi_\gamma)$. Denote $\mathcal{T} = \{\xi_\gamma = m_\gamma a_\gamma : \gamma \in \Gamma \cap P s N\}$.

<u>Ex.</u> If $G = SL(2,\mathbb{R})$ $\Gamma = SL(2,\mathbb{Z})$

$c \neq 0$ $\gamma = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \Gamma \setminus \Gamma \cap P$

$$\gamma = \underbrace{\begin{vmatrix} 1 & \frac{a}{c} \\ 0 & 1 \end{vmatrix}}_{n_1(\gamma)} \underbrace{\begin{vmatrix} sg(c) \end{vmatrix}}_{m_\gamma} \underbrace{\begin{vmatrix} |c|^{-1} & \\ & |c| \end{vmatrix}}_{a_\gamma} s \underbrace{\begin{vmatrix} 1 & d/c \\ 0 & 1 \end{vmatrix}}_{n_2(\gamma)} \qquad s = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

$$\chi_m\left(\begin{vmatrix} 1 & x \\ 0 & 1 \end{vmatrix}\right) = e^{2\pi i m x}$$

, thus get