Now $\psi_x(y) := \psi\left(\frac{y}{x}\right)$ is a smth approx of $\chi_{[x,2x]}$ (x > 0 small).

Now define:

(*) $$h_x(\nu) = \frac{M\psi_x(-\nu) - M\beta_x(-\nu)}{g(\nu)} + \frac{M\psi_x(\nu) - M\beta_x(\nu)}{g(-\nu)}$$

where $\beta_x \in C_c^\infty(0,\infty)$ has to be chosen conveniently.

(1) Need $(M\beta_x)^{(l)}(\zeta) = (M\psi_x)^{(l)}(\zeta)$ $\quad 0 \leq l \leq k_\zeta - 1$

$- k_\zeta =$ order of zero

for every $\zeta \in Z = \left\{ \begin{matrix} \text{zeroes of } g(\nu) \\ \text{in } [0,\rho] \end{matrix} \right\}$

Now $\exists B > 0$ s.t $a_\gamma^\alpha \leq B \ \forall \gamma \in \Gamma \setminus \Gamma \cap P$, then so, if choose β_x : $\operatorname{supp}(\beta_x) \subset (B, +\infty)$, the Kloosterman term is not influenced by β_x. It is possible to choose such β_x, satisfying (1)

One has

- $M\beta_x(\nu) = O\left(x^{\zeta_0}\left(1 + |\log x|^{k_\zeta - 1}\right)\left(1 + |\operatorname{Im} \nu|\right)^{-l}\right)$

for $\operatorname{Re}\nu \leq \sigma$, $|\operatorname{Im}\nu| \geq 1$, for any $l \geq 0$.

Here $\zeta_0 = \min\{\text{zeroes of } g(-\nu) \text{ in } (0,\rho]\}$.