$$\sum_{j\geq 1} k(\nu_j)\,\underbrace{\tilde{\varphi}_j(n)\,\tilde{\varphi}_j(m)}_{\substack{\text{Fourier coeffs}\\ \text{of } \varphi_j}} + \frac{1}{2\pi i}\int_{-\infty}^{+\infty} k(it)\,\underbrace{\tilde{E}(it,n)\,\tilde{E}(it,m)}_{\substack{\text{Fourier coeffs}\\ \text{of E.S}}}\,dt$$

$$= \underbrace{\delta_{m,n}\,\Delta(k)}_{\text{Delta term}} + \underbrace{K_{n,m}(Bk)}_{\text{Kloosterman term}}$$

$k(\nu)$ is a test fn (even, holo on $|\mathrm{Re}\,\nu| < \frac{1}{2}+\varepsilon$, with certain decay as $\mathrm{Im}\,\nu \to \infty$)

Bk is a Bessel transform

$K_{m,n}(\cdot)$ involves classical Kloosterman sums

If $m,n \in \mathbb{Z}\setminus\{0\}$, $c \in \mathbb{N}$, let

- $$S(m,n;c) = \sum_{\substack{a \bmod c\\ ad\equiv 1\ (c)}} e^{2\pi i\left(\frac{am+dn}{c}\right)}$$

One has Weil-Salié estimates: $S(m,n;c) = O\left(c^{\frac{1}{2}+\varepsilon}\right)$, $c \to +\infty$

Linnik conjecture: (~1962)
$$\sum_{0<c\leq X} \frac{S(m,n;c)}{c} = O(X^{\varepsilon}) \quad \forall \varepsilon>0,\ X\to+\infty$$