<u>Theorem</u> ([M-Wallach 90])

If $k(\nu)$ is even, holo on a strip $|\mathrm{Re}\,\nu| < \sigma$

- $|k(\nu)| \leq C\, e^{-\pi |\mathrm{Im}\,\nu|} (1 + |\mathrm{Im}\,\nu|)^{2+\delta}$ $\qquad (\sigma = \rho + \varepsilon)$

and $\chi, \chi' \in (\Gamma \cap N \backslash N)^\wedge$, then

- $$\int_{\mathcal{S}} k(\nu)\, d\sigma(\nu) = \alpha(\chi)\, \delta_{\chi,\chi'} \int_{\mathrm{Re}\,\nu = 0} k(\nu)\, \nu \sin \pi\nu \, d\nu$$

$$+ \frac{1}{2\pi i} \sum_{[\gamma \in]} S(a_\gamma m_\gamma)\, a_\gamma^{\rho} \int_{\mathrm{Re}\,\nu = 0} k(\nu)\, g(\nu)\, \tau_{\chi,\chi'}(m_\gamma a_\gamma, \nu)\, d\nu$$

- Here $d\sigma$ is a measure with support $\mathcal{S} \subset i[0,\infty) \cup (0, \rho)$

It involves Fourier coefficients $c_\chi(\varphi_j), c_{\chi'}(\varphi_j)$ and Fourier coefficients of Eisenstein series.

- $g(\nu)$ is a product of exponentials and (Γ-functions)$^{-1}$ without zeros on $\mathrm{Re}\,\nu \geqslant 0$

$$g(\nu) \sim ct\, \frac{M^{\mathrm{Re}\,\nu} \cdot e^{\pi |\mathrm{Im}\,\nu|}}{(1 + |\mathrm{Im}\,\nu|)^{2\,\mathrm{Re}\,\nu + \kappa}} \quad \text{as } |\mathrm{Im}\,\nu| \to \infty$$

$$(\kappa = \kappa(\sigma))$$

Ex $G = SO(n,1)_0$ $\qquad g(\nu) = ct\, \Gamma(\nu)^{-1}\, \Gamma(\nu + \rho)^{-1}$