Wish to apply the formula to the estimation of generalized Kloosterman sums. For this, ideally, one would like to put a smooth approximation of the characteristic fn of an interval in the Kloosterman term.

Obstacle: the transform

$$\bullet \quad k(\nu) \to \left(ma \to \int_{\mathrm{Re}\,\nu=0} k(\nu)\, g(\nu)\, \tau_{\chi,\chi'}(\nu, ma)\, d\nu\right)$$

is complicated to invert. It is of Bessel type for $SL(2,\mathbb{R})$, $SL(2,\mathbb{C})$. For $SO(n,1)$ $(n \geq 4)$, $SU(n,1)$ it is given by a complicated series. Positive side: $\tau(\nu,a)$ has an expansion

$$- \quad \tau(\nu,a) = a^{\nu+\rho} \sum_{j\geq 0} q_j(\nu)\, a^{j\alpha} \qquad q_1 \equiv 1$$

and $q_j(\nu)$ a rational fn, holo on $\mathrm{Re}\,\nu \geq 0$ + unif. bounded on vertical strips.

<u>Proposition</u>. $\tau(\nu, \overset{m}{a}) = \left(1 + O\left(\frac{a^\alpha}{1+|\nu|}\right)\right) a^{\nu+\rho}$, uniformly on vertical strips $-\varepsilon \leq \mathrm{Re}\,\nu \leq \rho + \varepsilon$