ISRAEL JOURNAL OF MATHEMATICS 86 (1994), 221–232

ON THE SURJECTIVITY OF SOME TRACE MAPS

BY

ELI ALJADEFF

Department of Mathematics
Technion — Israel Institute of Technology
32000 Haifa, Israel

ABSTRACT

Let K be a commutative ring with a unit element 1. Let Γ be a finite group acting on K via a map $t\colon \Gamma \to \mathrm{Aut}(K)$. For every subgroup $H \leq \Gamma$ define $\mathrm{tr}_H\colon K \to K^H$ by $\mathrm{tr}_H(x) = \sum_{\sigma\in H} \sigma(x)$. We prove

THEOREM: tr_Γ *is surjective onto* K^Γ *if and only if* tr_P *is surjective onto* K^P *for every (cyclic) prime order subgroup* P *of* Γ.

This is false for certain non-commutative rings K.

0. Introduction

Let K be a commutative ring with a unit element 1 and let Γ be a finite group acting on K via a morphism $t\colon \Gamma \to \mathrm{Aut}(K)$. For every subgroup H of Γ and $x \in K$ define the trace map $\mathrm{tr}_H\colon K \to K$ by

$$\mathrm{tr}_H(x) = \sum_{\sigma\in H} \sigma(x).$$

Clearly, the image of this map lies in K^H, the H invariant elements in K. In this paper I discuss the relation between the surjectivity of $\mathrm{tr}_\Gamma\colon K \to K^\Gamma$ and of $\mathrm{tr}_H\colon K \to K^H$ for various subgroups $H \leq \Gamma$. Note that tr_H is a K^H linear map, so that the surjectivity of tr_H onto K^H is equivalent to the existence of an element $x_H \in K$ with $\mathrm{tr}_H(x_H) = 1$. Using this, it is easily shown that the surjectivity of tr_Γ onto K^Γ implies the surjectivity of tr_H onto K^H for every subgroup H in Γ. (For, if $\{t_1, \dots, t_n\}$ are representatives of the left cosets of H

Received July 2, 1992 and in revised form March 21, 1993